Thus, maxima in the wall temperature and, accordingly, minima in the heat-transfer coefficient are observed in several sections of the tube on the graphs of $t_{W}=f(x / d)$ (Fig. 1a, b). The graphs of the dependence $t_{w}=f(q)$ show that the maxima in wall temperature along the length correspond to sections CD (Fig. 2). It follows from Fig. 1a, b that a maximum in the wall temperature is not observed in the readings of all the thermocouples located at different distances along the length of the tube. But the graph of $t_{W}=f(q)$ constructed for thermocouples located at different distances from the tube inlet (Fig. 4) shows that the sections $C D$ where $t_{W}$ grows with an increase in $q$ are obtained for all the thermocouples.

Consequently, it is probably more advisable to judge the mode of heat transfer not from the graph of $t_{W}=f(x / d)$, but from the dependence $t_{W}=f(q)$.

## NOTATION

$t_{W}, t_{l}$, temperatures of wall and liquid, ${ }^{\circ} \mathrm{C}$; $\mathrm{t}_{l}^{\mathrm{in}}$, liquid temperature at tube inlet, ${ }^{\circ} \mathrm{C} ; \mathrm{p}$, pressure, bars; $t_{c r}$, critical temperature, ${ }^{\circ} \mathrm{C}$; $\mathrm{p}_{\mathrm{cr}}$, critical pressure, bars; $\mathrm{q}_{\text {s }}$ heat flux density, $\mathrm{W} / \mathrm{m}^{2}$; $\rho \mathrm{w}$, mass flow rate, $\mathrm{kg} / \mathrm{m}^{2} \cdot \sec ; \tau$, time, sec; $\mathrm{t}_{\mathrm{m}}$, pseudocritical temperature.

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## SELECTION OF SIMILARITY CRITERIA IN

STUDYING THE EFFECT OF ROTATION ON
HEAT EXCHANGE IN TURBINE BLADES
V. A. Trushin

UDC 536.244:621

The supplementary factors affecting the nature of the flow and heat exchange in rotating turbine arrays and the supplementary criteria reflecting the effect of rotation on heat exchange in turbine blades on the gas and air sides are discussed.

By now rather extensive experimental data have been accumulated on the intensity of heat exchange between a gas and rotating turbine blades [1-7]. In [1,3] the experimental results are given without generalization, but it clearly follows from them that rotation not only considerably intensifies but also causes a redistribution of the heat-exchange coefficient over the contour of the profile in a rotor array. In [2] the experiments are generalized for conditions of nonisothermal flow in the interblade channels, while in [4, 5] they are generalized for cases which are close to isothermal. In $[6,7]$ the heat exchange was estimated by an indirect method based on measurements of the blade temperatures with subsequent inverse calculation. The effect of
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rotation on heat exchange in rotor blades was not detected in [6, 7], nor was the effect of the angles of attack on the heat-exchange intensity detected. On the one hand, the experimental results of [6, 7] indicate that the method used by the authors does not conform in accuracy to the stated problem, while, on the other hand, as was correctly remarked in [8], the identical values of the measured temperatures of blades being cooled in a static state and in rotation can also be explained by the mutually compensating effect of the intensification of heat exchange under the conditions of rotation both in the interblade channels and in the intrablade cooling channels.

An analysis of $[6,7]$ shows that the authors incorrectly and incompletely analyzed the equation of motion

$$
\begin{equation*}
\rho\left[\frac{\partial \vec{\omega}}{\partial \tau}+\vec{w}(\operatorname{grad} \vec{w})\right]=\rho \vec{T}-\operatorname{grad} p+\mu \nabla^{2} \vec{w} \tag{1}
\end{equation*}
$$

in application to a turbine stage, taking as the similarity criteria the relationship to one another of the projections of one and the same vector $\overrightarrow{\mathrm{T}}$ of forces of the same nature (mass forces), and they came to a doubtful conclusion concerning the choice of the determining parameters which take into account the effect of rotation on the heat exchange of rotor blades.

As is known [9], the nature and stability of motion in a boundary layer, and, consequently, the heat exchange, are determined by the relationship of the forces of different natures (inertial forces of motion, mass forces, viscosity, pressure) acting on a stream element. In a turbine array under the conditions of rotation supplementary forces appear in the stream, and it is just the relationship of these supplementary forces to the forces of viscosity and inertia which determines the supplementary similarity criteria of the processes of heat exchange of rotor blades [10].

The hydrodynamics and heat exchange on turbine blades are determined by a system of differential equations which includes the equation of motion (1), the heat-exchange equation

$$
\begin{equation*}
\alpha=\frac{\lambda}{\Delta T}\left(\frac{\partial T}{\partial n}\right)_{n=0}, \tag{2}
\end{equation*}
$$

the energy (heat-conduction) equation

$$
\begin{equation*}
\frac{D T}{d \tau}=\frac{\lambda}{c_{p} \rho} \nabla^{2} T, \tag{3}
\end{equation*}
$$

and the continuity equation

$$
\begin{equation*}
\frac{\partial \rho}{\partial \tau}+\operatorname{div}(\overrightarrow{\rho w})=0 . \tag{4}
\end{equation*}
$$

To make the heat-exchange phenomenon concrete the system of equations (1)-(4) is combined with uniqueness conditions which characterize the geometry of the turbine stage, the physical properties of the gases, the initial time conditions, and the boundary conditions (at the entrance and exit of the stage).

The supplementary forces and high-speed periodic unsteadiness of the stream which develop in the transition from stationary to rotating turbine arrays are taken into account only by the equation of motion (1). Therefore, it is sufficient to analyze only Eq. (1) to obtain the supplementary criteria reflecting the effect of the supplementary factors on the heat exchange of rotor blades.

In developing the supplementary criteria under the conditions of rotation in [6, 7] the authors relate the supplementary forces to the tangential force $T_{t}$ acting on the array of blades, assuming it to be a mass force. In reality, the force $T_{t}$ is the inertial force of the change in momentum of the stream in the tangential direction. The authors of $[6,7]$ relate the difference in centrifugal acceleration forces to the volume of gas in the interblade channel ( $\mathrm{F}_{\mathrm{i}}, \mathrm{c}^{l}$ ) and the force $\mathrm{T}_{\mathrm{t}}$ to the volume of the per-second flow rate of the gas $\left(\mathrm{Gv} / \mathrm{z}=\mathrm{w}_{\mathrm{a}} \mathrm{t} t\right)$. The requirement that all the forces be related to one and the same volume in the construction and analysis of the equation of motion is thereby violated. There are also other incompatibilities with the requirements of similarity theory in $[6,7]$.

The reduction of the equation of motion [1] to dimensionless form by the widely known method of scalar transformations takes it into the form

$$
\begin{equation*}
\frac{l_{0}}{w_{0} \tau_{0}}\left[\frac{\partial \overline{\vec{w}}}{\partial \bar{\tau}}\right]+[\overrightarrow{\vec{w}}(\operatorname{grad} \overrightarrow{\vec{w}})]=\frac{\overrightarrow{\vec{T}_{0}}}{w_{0}^{2}} l_{0}^{2}[\overrightarrow{\vec{T}}]-\left[\frac{\partial}{\partial \vec{n}}\left(\frac{\rho}{\rho w_{0}^{2}}\right)\right]+\frac{\mu}{\rho w_{0} l_{0}}\left[\nabla^{\stackrel{\rightharpoonup}{\vec{w}}}\right], \tag{5}
\end{equation*}
$$

where the parameters with the subscript 0 are the reduction scales, while the parameters referred to them are marked by an upper bar.

The complexes in front of the brackets in (5) reflect the ratios of the forces of various kinds acting on a unit mass of gas to the inertial forces of the moving stream.

The complex $l_{0} / \mathrm{w}_{0} \tau_{0} \equiv \mathrm{Sh}$, called the Strouhal number, reflects the effect on the flow and heat exchange of the high-speed periodic unsteadiness of the gas stream from the encounter between the rotor blades and the edge wakes of the guide blades; it is easily changed to the form

$$
\begin{equation*}
\mathrm{Sh}=\frac{u_{\mathrm{av}}}{w_{2} t_{\mathrm{n}}}, \tag{6}
\end{equation*}
$$

if as the characteristic time $\tau_{0}$ one takes the period in which a rotor blade travels the spacing between edge wakes ( $\tau_{0}=t_{\mathrm{n}} / \mathrm{u}_{\mathrm{av}}$ ) and if as the characteristic velocity one takes the relative velocity at the exit from the rotor blades $\left(w_{0}=w_{2}\right)$. As the characteristic dimension one can take the blade height ( $\left.l_{0}=l\right)$, and then $\overline{\mathrm{t}}_{\mathrm{n}}=$ $\mathrm{t}_{\mathrm{n}} / l$.

The complex $T_{0} l_{0} / W_{0}^{2}=S$ reflects the effect of the mass forces on the flow and heat exchange in the rotor array. As the scale $\overrightarrow{\mathrm{T}}_{\mathbf{0}}$ one can take the total mass force vector

$$
\begin{equation*}
\vec{T}_{0}=\vec{T}_{\text {c.ro }}+\vec{T}_{l . \text { ro }}+\vec{T}_{\text {cor.ro }}+\vec{T}_{\text {c.def }}-\vec{T}_{l . \text { def }}+\vec{T}_{\text {cor.def }} \tag{7}
\end{equation*}
$$

Consequently, the complex $S$ cen be represented by six components. The first three

$$
\frac{T_{\text {c.ro }} l_{0}}{w_{0}^{2}}=\mathrm{S}_{u}^{\prime}, \frac{T_{\text {cor.ro }} l_{0}}{w_{0}^{2}}=\mathrm{W}_{r . u}, \quad \frac{T_{. \text {ro }} l_{0}}{w_{0}^{2}}=\Delta T_{u}
$$

reflect the factor of rotation, while the other three

$$
\frac{T_{l . \text { def }} l_{0}}{w_{0}^{2}}=\Delta T_{\text {def }}, \quad \frac{T_{\text {c.def }} l_{0}}{w_{0}^{2}}=\mathrm{S}_{\mathrm{g}}^{\prime}, \frac{T_{\text {cor.def } l_{0}}}{w_{0}^{2}}=\mathrm{S}_{\mathrm{g}}^{\prime \prime}
$$

reflect the factor of deflection in the interblade channels.
The last two complexes $S_{g}^{\prime}$ and $S_{g}^{\prime \prime}$ were discussed in [11] for the case of axial turbines.
Thus, the effect of rotation on the flow and heat exchange in rotor arrays is determined by the complexes $\mathrm{Sh}_{\mathrm{u}}, \mathrm{S}_{\mathrm{u}}^{1}, \mathrm{~W}_{\mathrm{r} . \mathrm{u}}$, and $\mathrm{T}_{\mathrm{u}}$.

The complex

$$
\begin{equation*}
\mathrm{S}_{u}^{\prime}=\frac{T_{\mathrm{c}, \mathrm{r}} l}{w_{2}^{2}}=\frac{\left(\frac{l_{\mathrm{p}}^{2}}{r_{\mathrm{p}}^{2}}-\frac{u_{\mathrm{b}}^{2}}{r_{\mathrm{b}}}\right) l}{w_{2}^{2}}=\frac{4 u_{\mathrm{a}}^{2} \mathrm{v}}{w_{2}^{2} \vartheta^{2}} \tag{8}
\end{equation*}
$$

(where $\vartheta=d_{a v} / l$ ) corresponds to the criterion $S_{u}=u_{a v} / w_{2} \vartheta$ obtained in [11]; we will also apply it to radialaxial turbines.

In the complex

$$
\mathrm{W}_{r . u}=\frac{T_{\mathrm{cor}^{\mathrm{ron}}} l}{w_{2}^{2}}=\frac{2 \omega_{2}^{l} \tilde{\omega}_{r}^{l}}{w_{2}^{2}}
$$

the radial velocity $w_{r}$ for radial-axial turbines is made up of the radial component of the mean flow-rate velocity $\mathrm{w}_{\mathrm{m}}$ flow. r , the radial velocity of secondary flows $\mathrm{w}_{\mathrm{r}}$.sec, and the radial velocity from the counterrotation of the gas $w_{r}$.ro in an interblade channel of a rotor array with an angular velocity $-\omega$.

If for the radial rotation velocity one takes $\mathrm{w}_{\mathrm{r} . \mathrm{ro}} \cong \omega\left(\mathrm{t}_{\mathrm{W}} / 2\right)$ and for the radial velocity from the secondary flows [12] one takes

$$
w_{\mathrm{r} . \mathrm{sec}}=w_{\mathrm{av}}(0.3-0.4) \cong w_{2}(0.25-0.35)
$$

one obtains

$$
\begin{align*}
& \mathrm{W}_{\mathrm{r} . u}=\frac{2 \omega \omega \frac{t_{\mathrm{w}}}{2}}{w_{2}^{2}}+\frac{2 \omega(0.25-0.35) \omega_{2} l}{w_{2}^{2}}+\frac{2 \omega \omega_{\mathrm{m}} \text { flowal }{ }^{l}}{w_{2}^{2}}= \\
& =\frac{4 u_{\mathrm{av}}^{2} \mathrm{w}^{l}{ }^{2}}{w_{2}^{2} d_{\mathrm{av}} d_{\mathrm{av}}}+\frac{4 u_{\mathrm{av}} l}{w_{2} d_{\mathrm{av}}}(0.25-0.35)+\frac{4 u_{\mathrm{av}} \mathrm{w}_{\mathrm{m}} \mathrm{flow}{ }^{2}{ }^{l} .}{w_{2}^{2} d_{\mathrm{av}}} . \tag{9}
\end{align*}
$$

The second term on the right side of Eq. (9) will equal zero for radial turbines, while the third term will equal zero for axial turbines. The complex

$$
\begin{equation*}
\Delta \mathrm{T}_{u}=\frac{T_{l . \mathrm{ro}} l}{w_{2}^{2}}=\frac{\frac{u_{\mathrm{av}}^{2}}{r_{\mathrm{av}}} \beta \Delta T l}{w_{2}^{2}} \tag{10}
\end{equation*}
$$

is changed to the form ( $\beta=1 / \mathrm{T}$ for an ideal gas)

$$
\begin{equation*}
\Delta \mathrm{T}_{u}=\frac{2 u_{\mathrm{av}}^{2}}{w_{2}^{2}} \frac{l}{d_{\mathrm{av}}} \frac{\Delta T}{T} \tag{11}
\end{equation*}
$$

and can also be used for radial-axial turbines.
It should be noted that for a compressible gas the Mach number M and the adiabatic index k , which reflect the effect of compressibility on the heat exchange in turbine arrays, are obtained from the second term on the right side of Eq. (5) :

$$
\begin{equation*}
\frac{p}{\rho w_{0}^{2}}=\frac{k p}{k p w_{0}^{2}}=\frac{a^{2}}{k w_{2}^{2}}=\frac{1}{k \mathrm{M}^{2}} . \tag{12}
\end{equation*}
$$

As is known, the complex $\mu / \rho \mathrm{w}_{0}^{2} l_{0}=\mu / \rho \mathrm{w}_{2} l=1 / \mathrm{Re} l$ [see (5)] is the inverse Reynolds number.
Thus, the ratio of the heat-exchange coefficient in a blade array on the gas side under conditions of rotation to that for a stationary array will be determined by the quantity

$$
\begin{equation*}
\varepsilon_{\mathrm{g}}=\frac{\alpha_{\mathrm{g} . \mathrm{ro}}}{\alpha_{\mathrm{g} . \mathrm{st}}}=f\left(\mathrm{Sh}_{u}, \mathrm{~S}_{u}^{\prime}, \mathrm{W}_{r . u}, \Delta \mathrm{~T}_{u}\right) . \tag{13}
\end{equation*}
$$

For the cooling channels the equation of motion in dimensionless form is written similarly to (5) and the intensification of heat exchange in the transition from stationary to rotating blades is determined by the same criteria, but calculated with respect to scalar quantities referred to the cooling air (which is indicated below by the subscript a).

We obtain

$$
\begin{equation*}
\mathrm{Sb}_{u \cdot \mathrm{a}}=\frac{u_{\mathrm{na}, \mathrm{a}} l_{\mathrm{a}}}{w_{2} t_{\mathrm{n}} \mathrm{a} \mathrm{a}}=\frac{k_{1} u_{\mathrm{av}, \mathrm{a}} l_{\mathrm{a}}}{w_{2 \mathrm{az}} t_{\mathrm{n} \cdot \mathrm{a}}} \tag{14}
\end{equation*}
$$

$\left(k_{1}=u_{n, a} / u_{\text {av.a }}\right.$ is determined from the geometry of the rotor).
The criterion which allows for the effect on the heat exchange of the nonuniformity of the centrifugal forces over the volume of the channel in the isothermal stream takes the form

$$
\begin{equation*}
S_{\mathrm{u} . \mathrm{a}}^{\prime}=\frac{4 u_{\mathrm{av}, \mathrm{a}}^{2} l_{\mathrm{a}}^{2}}{w_{2 \mathrm{a}}^{2} d_{\mathrm{av}}^{2}} \tag{15}
\end{equation*}
$$

and is obviously applicable to channels of arbitrary configuration.
The complex reflecting the effect of Coriolis forces has the form
( $\mathrm{k}_{2}=\mathrm{w}_{\mathrm{r} . \mathrm{a}} / \mathrm{w}_{2 \mathrm{a}}$ is determined from the geometry of the rotor).
For radial channels, where $W_{r . a}=w_{2 a}$, this complex changes to the form ( $\mathrm{k}_{2}=1$ )

$$
\begin{equation*}
\left(\mathrm{W}_{\mathrm{r}, \mathrm{u}, \mathrm{a}}\right)_{\mathrm{rad}}=\frac{4 u_{\mathrm{av}} l_{\mathrm{a}}}{w_{2 \mathrm{a}} d_{\mathrm{av}, \mathrm{a}}} . \tag{17}
\end{equation*}
$$

A comparison of (16) and (17) with (15) shows that the criterion $W_{r . u . a}$ reduces to the complex $S_{u . a}^{\prime}$, namely,

$$
\begin{equation*}
\mathrm{W}_{\text {r. } \mathrm{u} \mathrm{a}}^{2}=4 k_{\mathrm{z}}^{2} \mathrm{~S}_{\mathrm{u}, \mathrm{a}}^{\prime}, \tag{18}
\end{equation*}
$$

i.e., the complex $S_{u . a}^{1}$ allows for the effect of both the centrifugal and Coriolis forces on the heat exchange in the cooling channels.

The complex allowing for the effect on the heat exchange of the nonisothermal nature of the stream in a cooling channel has the form

$$
\begin{equation*}
\Delta \mathrm{T}_{\mathrm{u} . \mathrm{a}}=\frac{\frac{u_{\mathrm{av}, \mathrm{a}}^{2}}{r_{\mathrm{av}, \mathrm{a}}} \beta_{\mathrm{a}} \Delta \mathrm{~T}_{\mathrm{a}} i_{\mathrm{a}}}{w_{\mathrm{za}}^{2}}=\frac{2 u_{\mathrm{av}, \mathrm{a}}^{2} d_{\mathrm{a}} \Delta \mathrm{~T}_{\mathrm{a}}}{w_{2 \mathrm{a}}^{2} d_{\mathrm{a}} \mathrm{~T}_{\mathrm{a}}} \tag{19}
\end{equation*}
$$

and is applicable both to axial and to radial turbines.
The intensification of heat exchange in the cooling channels in the transition from stationary to rotating blades can be generalized in the form of the critical dependence

$$
\begin{equation*}
\varepsilon_{\mathrm{a}}=\frac{\alpha_{\mathrm{a} \cdot \mathrm{ro}}}{\alpha_{\mathrm{a}, \mathrm{st}}}=\varphi\left(\mathrm{Sh}_{\mathrm{u} \cdot \mathrm{a}}, \mathrm{~S}_{\mathrm{u} \cdot \mathrm{a},}^{\prime} \Delta \mathrm{T}_{\mathrm{u} \cdot \mathrm{a}}\right) \tag{20}
\end{equation*}
$$

One must also conclude that:

1. The laws of heat exchange in turbine rotor blades on the gas side require further refinement. It is necessary to introduce the supplementary criteria $\mathrm{Sh}_{\mathrm{u}}, \mathrm{S}_{\mathrm{u} . a}^{\mathrm{a}}, \mathrm{W}_{\mathrm{r} . \mathrm{u}}$, and $\Delta \mathrm{T}_{\mathrm{u}}$, reflecting the effect of highspeed periodic unsteadiness and of centrifugal, Coriolis, and lifting forces, respectively, on the nature of the flow and heat exchange on rotating blades. It is necessary to refine the criterial equations for unsteady conditions at increased Mach numbers.
2. Data on heat exchange in the cooling channels of rotor blades with air cooling are practically absent. The data available in [6, 7] are obtained by indirect means, and besides, as indicated in [8], they must be applied with caution. Direct studies of this problem are urgently needed. The criterial equation for the calculation of the coefficients of heat exchange in rotating intrablade cooling channels should contain the criteria $\mathrm{Sh}_{\mathrm{u} . \mathrm{a}}, \mathrm{S}_{\mathrm{u} . \mathrm{a}}^{\mathrm{a}}$, and $\Delta \mathrm{T}_{\mathrm{u}}$ a which reflect the effect of high-speed periodic unsteadiness, centrifugal and Coriolis forces, and lifting forces, respectively, on the hydrodynamics and heat exchange in them.
3. It should be noted that in this report it was not possible to allow for and find the criteria reflecting the effect of the initial turbulence of the stream on the heat-exchange intensification $\epsilon$ in the transition from static to rotating arrays or on the turbulence intensification in this case.
4. Such an important factor as the site of the transition from a laminar to a turbulent boundary layer along the profile of a rotor blade remains unaccounted for. The correct choice of the transition site provides for better agreement between the calculated and experimental heat-exchange coefficients $\alpha_{\mathrm{g}}$ [14], but in the present report it did not seem possible to find the criterion reflecting the effect of rotation on the site of the transition from a laminar to a turbulent boundary layer.

## NOTATION

$t_{n}, u_{\text {av }}$, spacing of nozzle blades and circular velocity of rotor blades at mean diameter of turbine rotor, respectively; $\vec{T}_{c . r o}, \vec{T} l$.ro, $\vec{T}_{c o r . r o, ~ v e c t o r s ~ o f ~ c e n t r i f u g a l, ~ l i f t i n g, ~ a n d ~ C o r i o l i s ~ f o r c e s ~ d u e ~ t o ~ r o t a t i o n ; ~}^{\text {, }}$ $\overrightarrow{\mathrm{T}}_{\mathrm{c} . \text { def, }}, \overrightarrow{\mathrm{T}}_{l}$.def, $\overrightarrow{\mathrm{T}}_{\text {cor.defs }}$ vectors of centrifugal, lifting, and Coriolis forces due to deflection of stream in an interblade channel; $u_{p}, u_{b}$ and $r_{p}, r_{b}$, circular velocity and radius of periphery and base of blades; $l$, blade length, $d_{a v}$, average diameter of blading; $t_{w}$, spacing of working array at average diameter; $u_{n . a}$, circular velocity of inlets of blade cooling channels having a supply of cooling air with preliminary swirling in a special nozzle apparatus [13]; $\mathrm{t}_{\mathrm{n} .}$, nozzle spacing of air nozzle apparatus; $\mathrm{w}_{2}$, relative stream velocity at exit from rotor array; $a$, velocity of sound; $k$, adiabatic index; $\alpha_{g . r o}, \alpha_{g, s t}, \alpha_{a, ~ r o}, \alpha_{a . s t}$, rotary and static coefficients of heat exchange on gas and air sides; wa, axial component of relative velocity in rotor array; $z$, number of rotor blades; $F_{i}$.c, frontal area of an interblade channel of rotor; $G, v$, flow rate of gas through rotor array and its specific volume; $n$, direction of normal; $T, P, \rho, \mu$, temperature, pressure, density, and viscosity, respectively; $\tau, \lambda$, coefficient of thermal conductivity; $c_{p}$, heat capacity.

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## ACOUSTIC DISPERSION IN RAREFIED GASES

V. A. Bubnov

UDC 534-13:532.51

The problem of acoustic dispersion in rarefied gases is solved on the basis of the hydrodynamical equations of Predvoditelev. The theoretical equation is compared with the experiments of Greenspan for five monatomic gases. Theory and experiment are compared up to a Knudsen. number of order unity.

## 1. On the Nonideal Continuity Parameter

In 1948 A. S. Predvoditelev described a technique for improving the Navier-Stokes equations in application to problems in which the hydrodynamic velocity gradient is related to the path traversed by the molecules between collisions. This technique is based essentially on the Maxwell transport equation and a more precise hypothesis regarding the relationship between the hydrodynamic flow velocity and the transport velocities of two colliding molecules. The indicated relationship must be determined in transforming from the Maxwell transport equation to the continuum equations.

If the most general assumptions are advanced with regard to the transport velocities of two colliding molecules, the equations for the hydrodynamic stresses have the form [2]

$$
\left.\begin{array}{c}
\rho \bar{\xi}_{i}^{2}=p+\frac{A_{1}}{3 A_{2}} \rho\left(v_{2 i}^{2}-v_{\mathrm{Li}}^{2}\right)-2 \mu\left(\frac{\partial v_{i}}{\partial x_{i}}-\frac{\gamma-1}{2} \operatorname{div} \mathrm{~V}\right) \\
\rho \overline{\bar{\xi}_{i j} \bar{j}}=-\mu\left(\frac{\partial v_{i}}{\partial x_{j}}+\frac{\partial v_{j}}{\partial x_{i}}\right)+\left(v_{2 i} v_{2 j}-v_{1 i} v_{1 j}\right) ; \quad i=1,2,3,  \tag{1.1}\\
j=1,2,3 .
\end{array}\right\}
$$

When the transport velocities of the two colliding molecules are equal, i.e., when $\mathrm{v}_{2 \mathrm{i}}=\mathrm{v}_{1 \mathrm{i}}=\mathrm{v}_{\mathrm{i}}$, Eqs. (1.1) go over to the expressions derived in the theory of the Navier-Stokes equations. Equations (1.1) can be used, however, to obtain the more complete Predvoditelev equations in the form [2]

$$
\begin{equation*}
\rho \frac{d v_{i}}{d t}+\sum_{i} \frac{A_{1}}{3 A_{2}} \cdot \frac{\partial}{\partial x_{i}}\left[\rho\left(v_{2 i} v_{2 j}-v_{1 i} v_{1 j}\right)\right]=-\frac{\partial p}{\partial x_{i}}+\mu\left[\nabla^{2} v_{i}+(2-\gamma) \frac{\partial}{\partial x_{i}} \operatorname{div} \mathbf{V}\right] \tag{1.2}
\end{equation*}
$$

It is important to note that in the Maxwell calculations $(2-\gamma)=1 / 3$, but this expression is valid only for a monatomic gas. Consequently, this restriction of Maxwell is tacitly implicit in the adiabatic equation for a monatomic gas. The latter fact was also first noted by Predvoditelev [1].

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